THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH3070 Introduction to Topology 2017-2018
Tutorial Classwork 5

1. Let X be a topological space and $\sim$ be an equivalent relation. The quotient topology of the space $X/\sim$ is defined by

$$\mathfrak{T}_{\text{quot}} = \{U \subset X/\sim \quad | \, \pi^{-1}(U) \in \mathfrak{T}_X\}$$

   Show that $\mathfrak{T}_{\text{quot}}$ is a topology on $X/\sim$.

2. Let X be a topological space and $\sim$ be an equivalent relation. Suppose that the quotient space $X/\sim$ with quotient topology is Hausdorff. Is X necessarily Hausdorff?

   (Hint: The equivalent relation may identify two non-separated points together.)

3. * Consider an equivalent relation $\sim$ on $\mathbb{R}^2$ defined by

$$(x_1, y_2) \sim (x_2, y_2) \iff y_1 - x_1^2 = y_2 - x_2^2$$

   Show that $(\mathbb{R}^2/\sim, \mathfrak{T}_{\text{quot}})$ is homeomorphic to $(\mathbb{R}, \mathfrak{T}_{\text{std}})$.